

Combinatorics: Summary and Examples

1. Introduction

“Combinatorics” or Enumeration is about determining the number of elements in various types of sets. Enumeration techniques are useful in particular for the calculation of probabilities i.e. in order to quantify the chance of occurrence of a wide range of random events.

2. Enumeration cases and formulas

A summary table of formulas is provided further below that gives the main enumeration cases and corresponding formulas.

A typical problem of enumeration is to find out how many different sets can be formed when selecting p objects among a total of n distinct objects. In order to analyze such an enumeration problem, two questions need to be addressed:

- With or Without repetition: can objects be repeated in the set or conversely should each object be selected only once ?
- With or Without order: Are the sets ordered or conversely is the order of objects of no importance ?

Summary of Enumeration formulas

Number of ways of selecting p elements among n

Illustration: selection of 2 letters among A, B, C

	WITHOUT REPETITION	WITH REPETITION
WITHOUT ORDER	<i>Combinations without repetition</i> $C_n^p = \binom{n}{p} = \frac{n!}{(n-p)! p!}$	<i>Combinations with repetition</i> $C_{n+p-1}^p = \binom{n+p-1}{p} = \frac{(n+p-1)!}{(n-1)! p!}$
	$C_3^2 = \frac{3!}{1! 2!} = 3$ <p>AB-AC-BC</p>	$C_4^2 = \frac{4!}{2! 2!} = 6$ <p>AA-AB-AC-BB-BC-CC</p>
WITH ORDER	<i>Arrangements without repetition</i> $A_n^p = \frac{n!}{(n-p)!}$	<i>Permutations ($p = n$)</i> $A_n^n = n!$
	$A_3^2 = \frac{3!}{1!} = 6$ <p>AB-AC-BA-BC-CA-CB</p>	$3^2 = 9$ <p>AA-AB-AC-BA-BB-BC-CA-CB-CC</p>

3. Example problems with solutions

Give it a try with the following problems :

1. Horse racing: number of possibilities for the first 3 horses in a race of 10 ?
2. Three dice, case 1: How many results are possible when rolling 1 dice 3 times in a row ?
3. Three dice, case 2: How many results are possible when rolling 3 indistinguishable dice simultaneously ?
4. Lottery: number of possible outcomes when drawing 6 balls out of 49 ?
5. Domino pieces: how many different domino pieces are there, given that each square has 1 to 6 spots or is blank ?
6. Seating people at a table: number of ways to seat 8 people around a table ?
7. Poker: how many poker hands are possible when 5 cards are dealt from 52 ?

8. Combination locker: number of possible codes of a 3 wheel combination locker, each wheel with digits 0 to 9 ?

Solutions: 1: 720, 2: 216, 3: 56, 4: 13,983,816, 5: 28, 6: 40,320, 7: 2,598,960, 8: 1000